This page is for

Teachers

Parents and carers

Digital Technologies @ Home
Unplugged activities for students

This activity is for: Years 7-8, 9-10

Hamiltonian and Eulerian Paths and Circuits

With thanks to the Girls' Programming Network for providing this content.

This activity teaches…

This is an introduction to Graph Theory problems and their real world applications by looking at Hamiltonian and Eulerian (pronounced "oil-air-ian") Paths and Circuits.

It helps to teach problem specification by showing scenarios broken down to 2 main components - the places to visit and the paths between them - as a diagram. Being presented with the visual map of the problem, students learn how to use it as a tool to find solutions to different scenarios that present themselves. It requires 'trial and error' solutions, introducing algorithmic filters to solutions later on.

It is targeted towards secondary students in years 7-10 and is expected to take 1.5 to 2 hours.

You will need…

Paper, pencil and eraser (Optional: a plastic sleeve and whiteboard marker. Place the printed graph question in the sleeve and use the whiteboard marker on the plastic so that it can be erased easily.)

Getting started (read this with your child):

Imagine you are trick-or-treating around your neighbourhood. You have to get to each house without stopping at any of the houses twice and you need to start and finish at your own house. This is an example of a Graph Theory problem that needs solving! What you need is called **a Hamiltonian circuit**: it's a path around the suburb that stops at each house once and gets you back home.

Now imagine you are in charge of re-paving the roads in your neighbourhood. You need to cover all of the roads, but you can't go over a road twice because the road will still be wet and your paving machine will get stuck. This is also a graph theory problem, and you're going to learn to find the solution.

Step by step

Work through the explanation sheets, discussing the rules of different paths and circuits. Work through the first map together, finding the different paths and circuits. (If you want to draw on the maps use pencil and eraser or the plastic sleeve option detailed above). There are multiple answers to many of these graphs.

Make sure your child understands the difference between the different routes they are expected to find. Then give your child time to work through the later options by themselves.

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This page is for

Students

Graph Theory Circuits

Learn how to find Hamiltonian and Eulerian Circuits for different maps!

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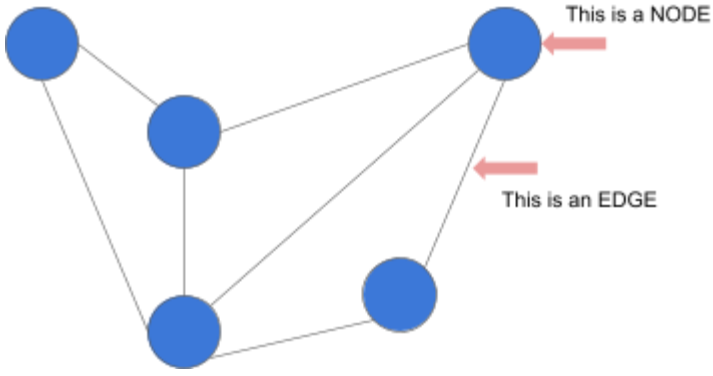
Graph theory problems are all around us. They look at the different ways you can move around places and objects, depending on what you need to do. The first thing to learn is how to read the graphs you will find in the puzzles.

These graphs don't have an x and y axis, they are made up of **Nodes** and **Edges.** The **nodes represent places to visit** and the **edges represent the different ways to get to the nodes**. You could think of it like houses and the streets that connect them.



Seems simple right? It's designed to be! Using these graphs we can break down what look like complex problems into easier to manage graphs.

We are going to look for ways to move around these graphs that solve problems. Specifically, Hamiltonian and Eulerian paths and circuits. The different types solve different problems, one is about visiting each node (Hamiltonain) the other is about visiting each edge (Eulerian).

The last thing to learn is that we are working with **undirected graphs**. The means you can go either way up or down the edges. A directed graph would have arrows on the edges that you would have to follow.

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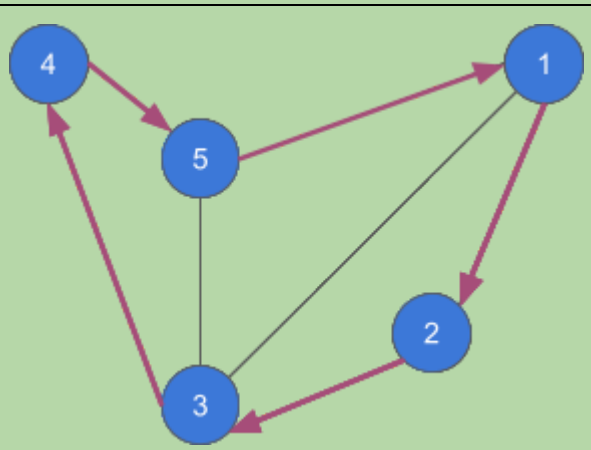
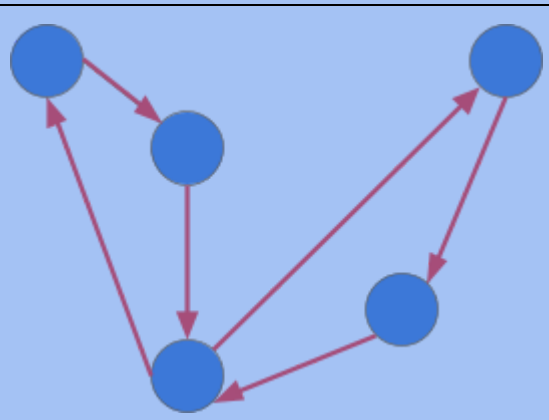
For each undirected graph question, there are 4 potential puzzles to find:

Hamiltonian Path - Visit every each **node** exactly once - Finish anywhere	**Eulerian Path** - Visit each **edge** exactly once - Finish anywhere
Hamiltonian Circuit - Visit every each **node** exactly once - Finish back where you start	**Eulerian Circuit** - Visit each **edge** exactly once - Finish back where you start

Not every graph has a Hamiltonian or Eurlerian property!
You might not be able to find all 4 puzzles in all the graphs.

Hamiltonian Circuit Example	Eulerian Circuit Example
	
A Hamiltonian circuit solution will visit each node and **must finish at the start node.** If it was a path, it could finish at any node.	An Eulerian circuit solution will visit each edge and **must finish at the start node.** If it was a path, it could finish at any node.
Hamiltonian Path Example	**Eulerian Path Example**

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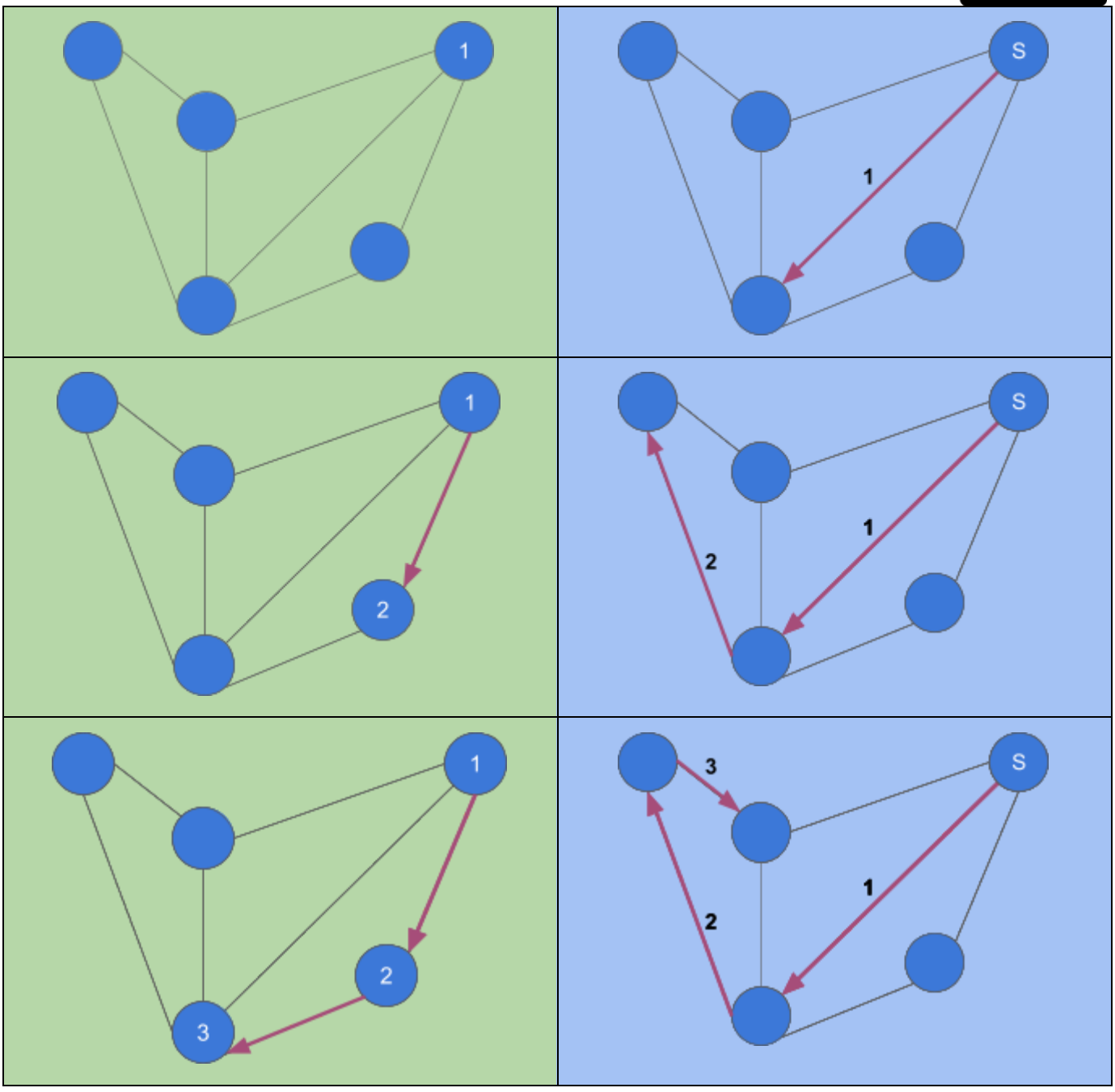

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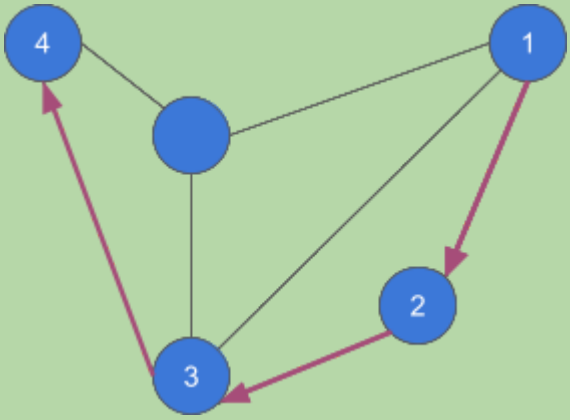
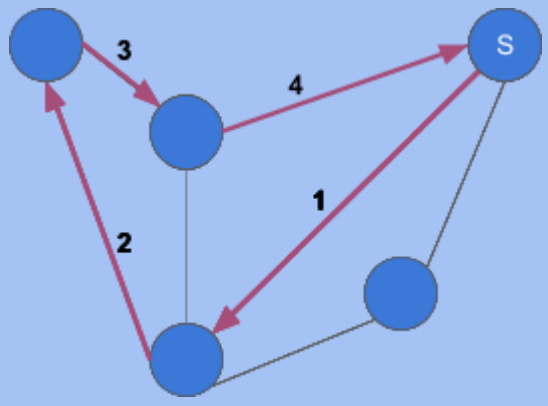
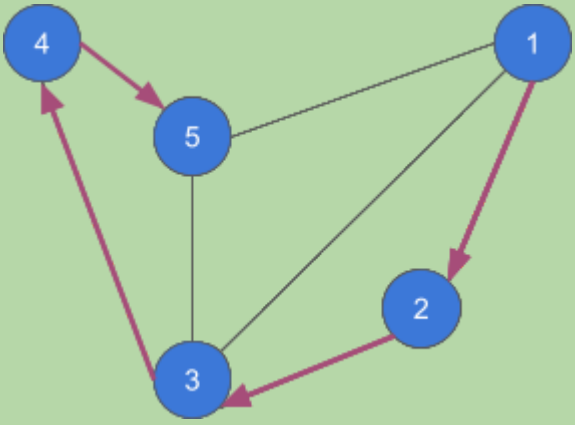
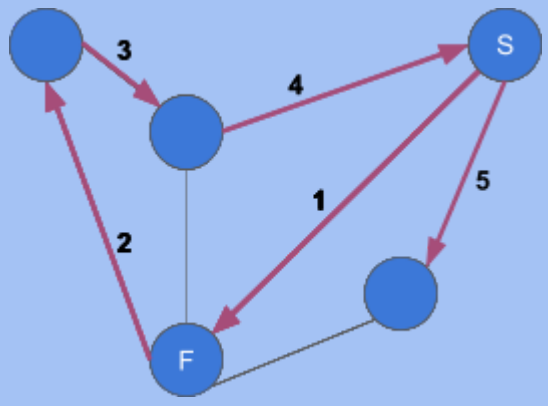
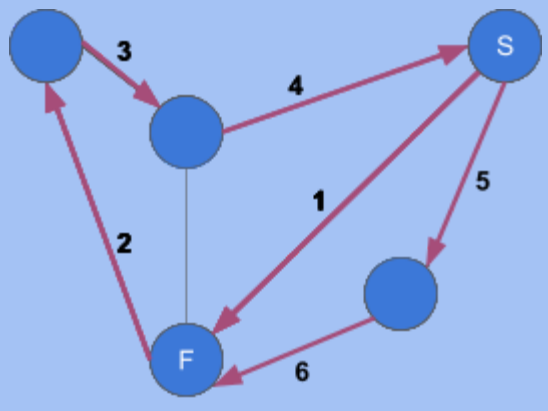






- All nodes visited
- No need to get to start node





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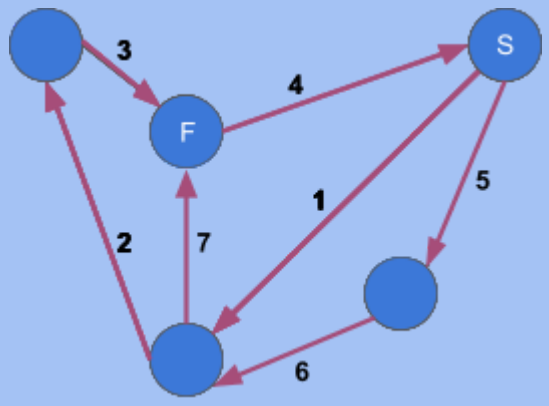

- All paths visited
- No need to get back to start node

You can choose to start at any node, unless the question tells you specifically where to start.

Something interesting to remember, **if a graph has an Eulerian circuit, you can start at any node and complete the circuit.**

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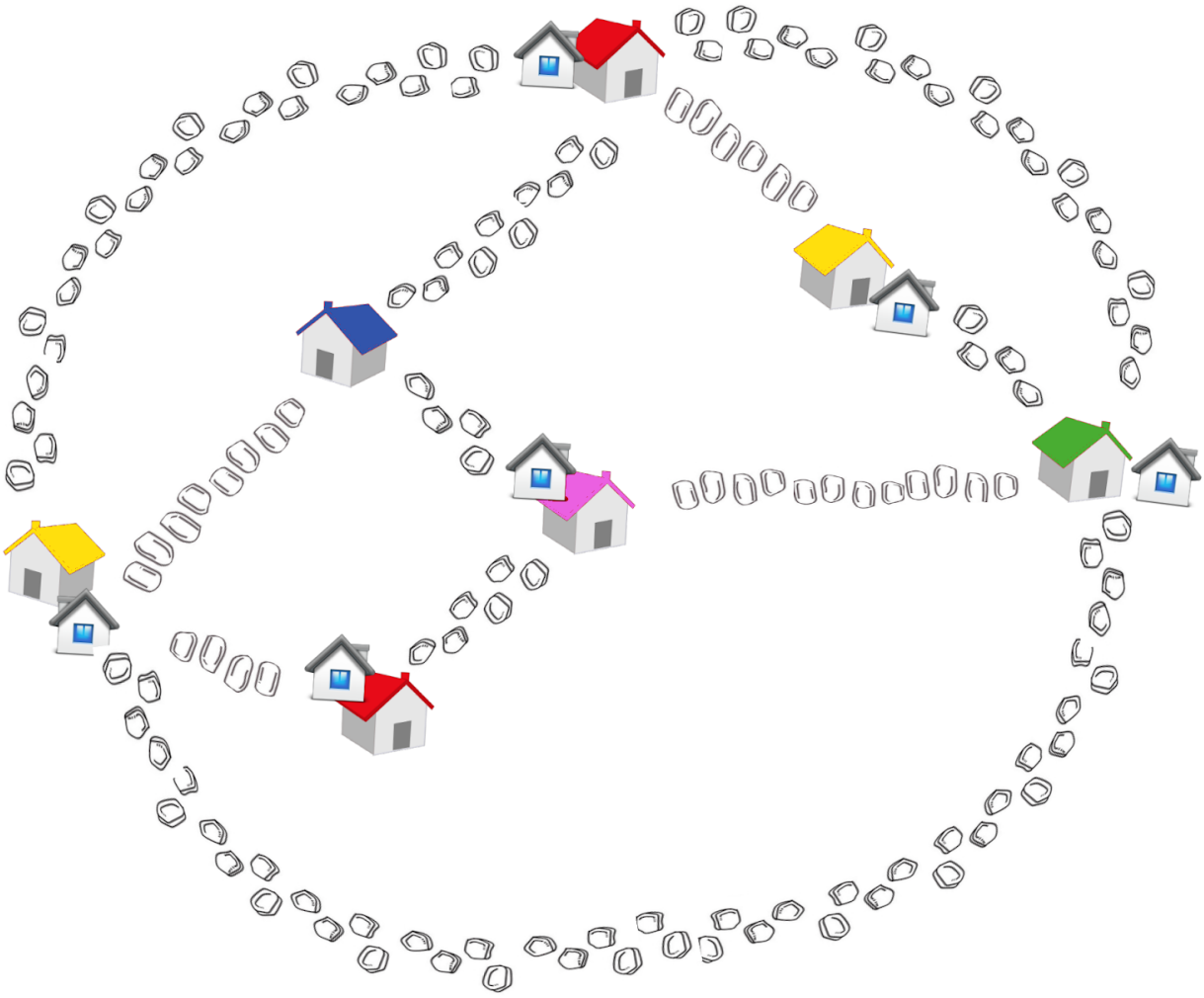
Puzzle 1!

You are the bus driver of the school bus, you drive around the neighbourhood and pick up students and take them to school each morning. Starting from the Green School House, find a route that visits each house node exactly once. You can finish anywhere.

1. Which kind of path do you need to find?
2. Find a path
3. Find a circuit that starts and finishes at the green school house.

Now imagine you are in charge of drawing maps for the suburb. You need to go down every street in the suburb and write their names down for the map. You are trying to be as efficient as possible, so you don't want to go over any streets twice.

4. Which kind of path do you need to find?
5. Find a path that goes over every street exactly once. You should start from the blue house but you can finish at any house. (This one can be tricky).



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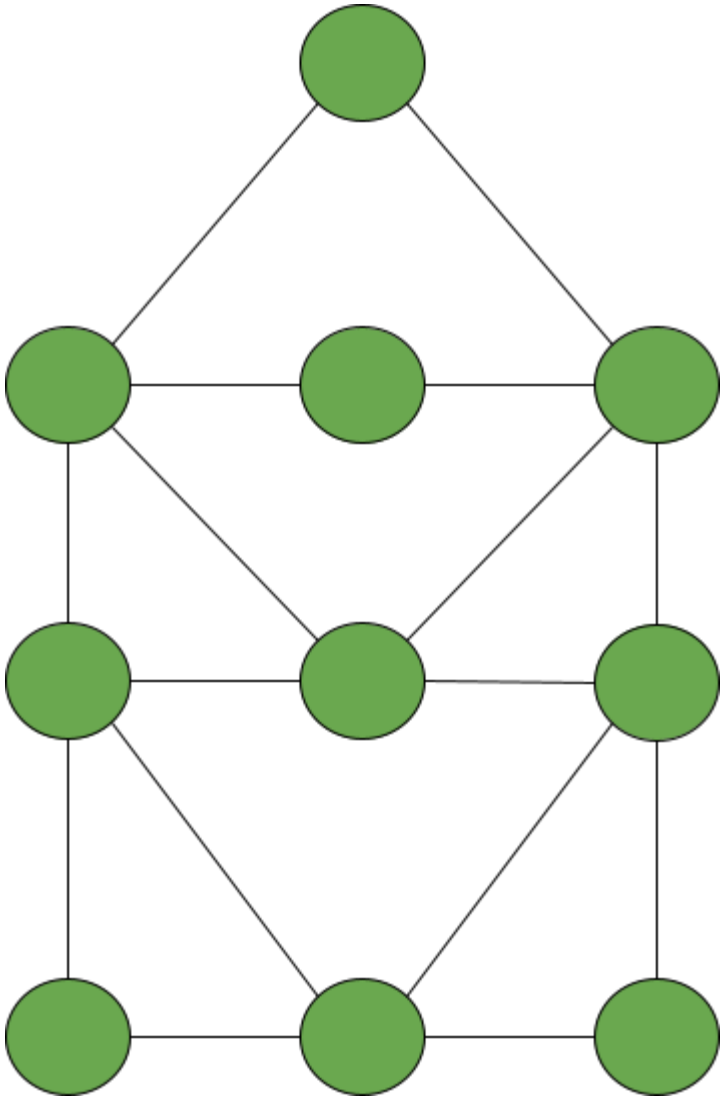
Puzzle 2!

It's time to paint the fences in the garden! This graph represents the fences (edges) and plants (nodes) in a garden. You have to find a path to paint every fence, you only have enough paint for 1 coat each, so you can visit each edge only once.

1. Which kind of path through the graph are you looking for?
2. Find a path that paints each edge exactly once, you can start anywhere.
3. Can you find a circuit? (It might not have one)

Now that all the fences are freshly painted, you should make sure the plants all have fertiliser. You only have enough fertiliser to visit each plant once.

4. What kind of path are you looking for?
5. Find a path that gets to every plant exactly once, starting anywhere.
6. Can you find a circuit? (It might not have one)



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Puzzle 3!

This graph represents the classrooms (nodes) at school and the paths (edges) that connect them together.

At school your teacher asks you to go along all the paths to make sure they are clean and tidy, but you only have a few minutes before the start of class so you can't go over any twice.

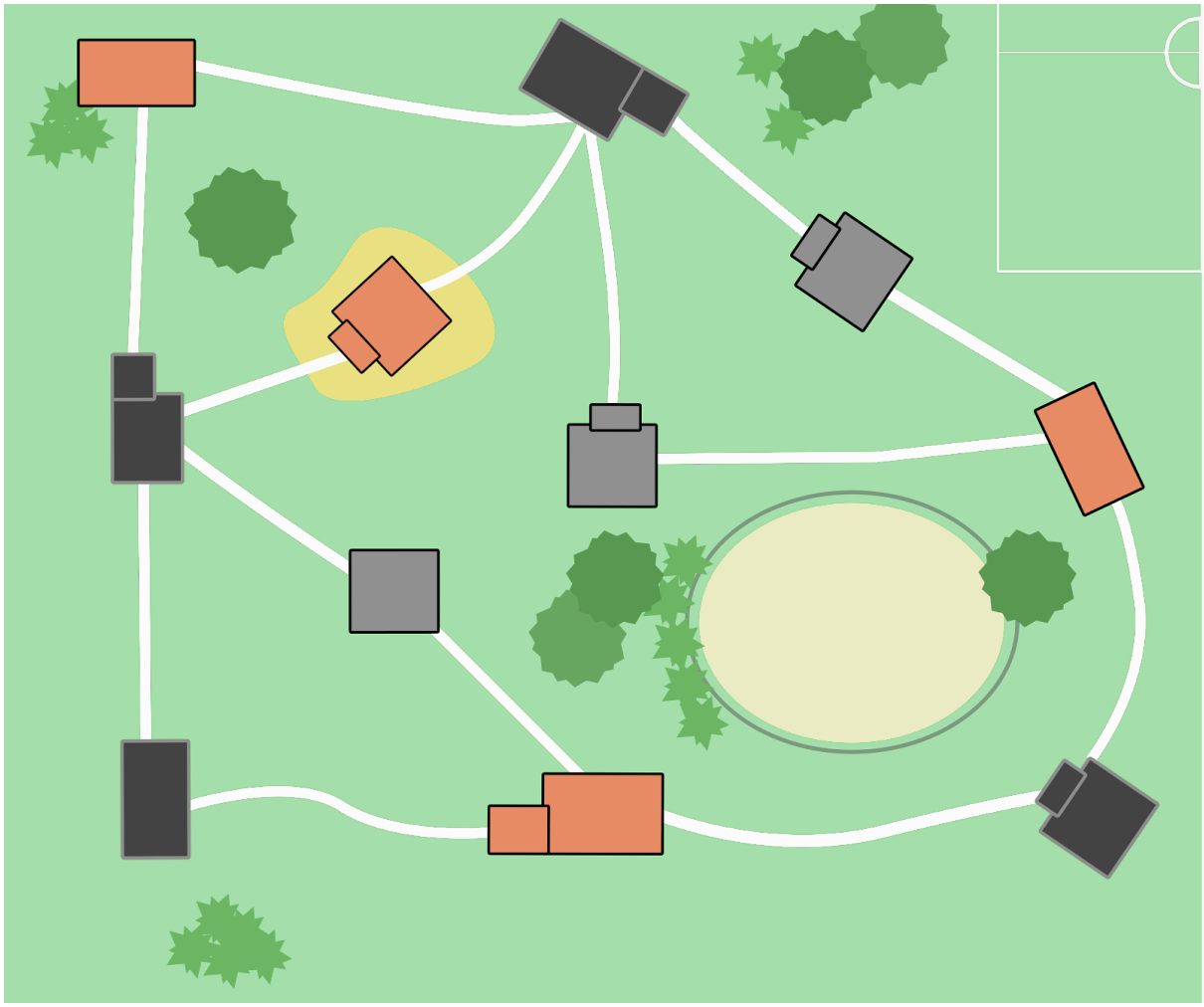
1. What kind of path are you looking for?
2. Can you find a path through the graph?
3. Can you find a circuit? (It might not have one)

You are in charge of collecting the recycling bins in all of the classrooms, you need to visit each room but only once!

4. What kind of path are you looking for?
5. Can you find a Path? (It might not have one)
6. Can you find a circuit? (It might not have one)

Hint question!

7. Adding one extra edge will make a circuit possible. Can you find which one? (Have a look at the next question for some guidance)



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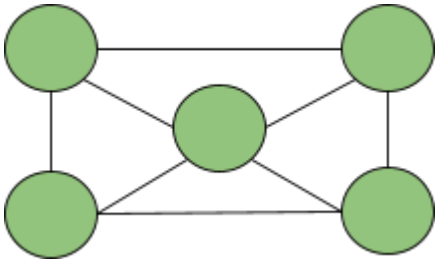
Bonus Puzzle! Eulerian quickfire round.

Looking for the paths and circuits can take a while, especially if there isn't one! Luckily, there are a few rules we can use to quickly check for Eulerian paths/circuits in a graph without having to trace any routes along the graphs.

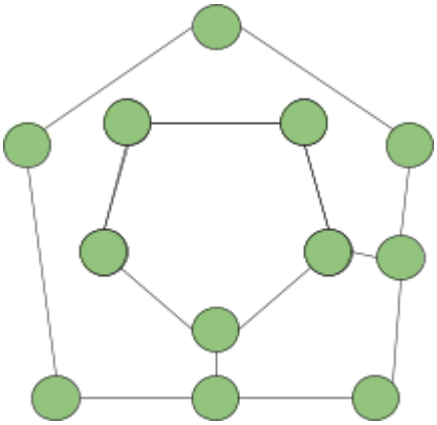
For each undirected graph:

- An Eulerian Circuit is only possible if **every node has an even number of edges.**
- An eulerian Path is only possible if **every node has an even number of edges or exactly 2 nodes have an odd number of edges.**

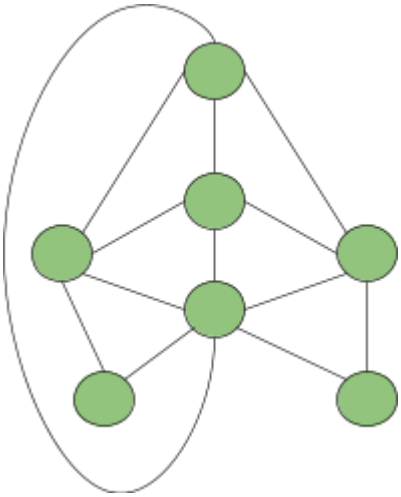
Using those rules, can you check for Eulerian paths and circuits in the graphs below using just the number of edges?



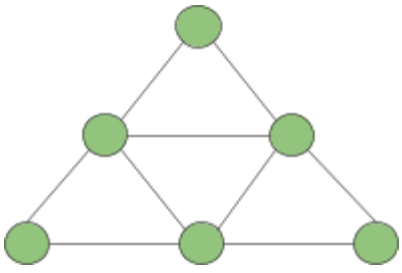
Path?: Y / N
Circuit? Y / N



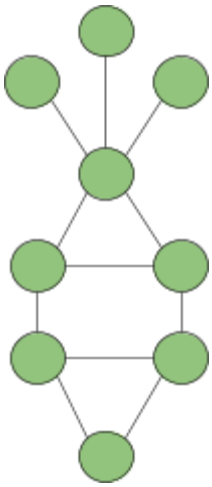
Path?: Y / N
Circuit? Y / N



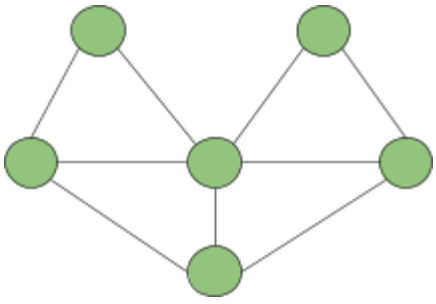
Path?: Y / N
Circuit? Y / N



Path?: Y / N
Circuit? Y / N



Path?: Y / N
Circuit? Y / N



Path?: Y / N
Circuit? Y / N

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This page is for

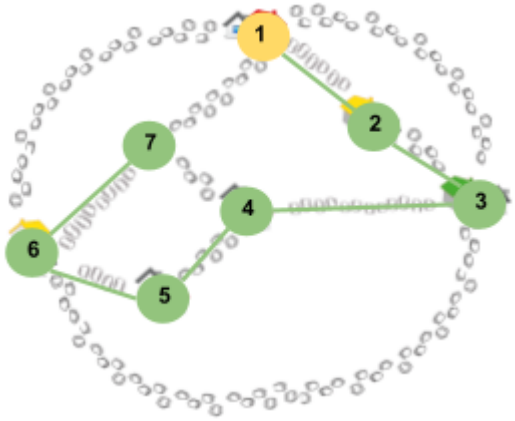
Parents and carers

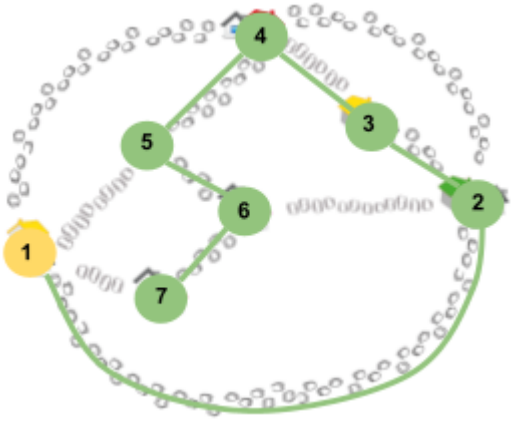
Answer key

Choose if you want to print this for your kids or keep it to yourself!

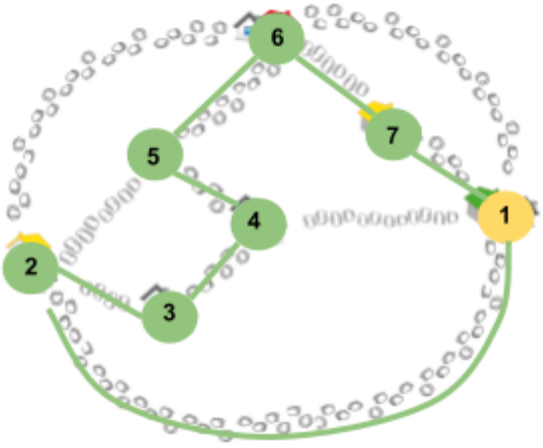
Puzzle 1

1. **A Hamiltonian Path will reach all the houses exactly once.**
2. Find a **Hamiltonian** path. **There are multiple answers, 2 examples are below.**

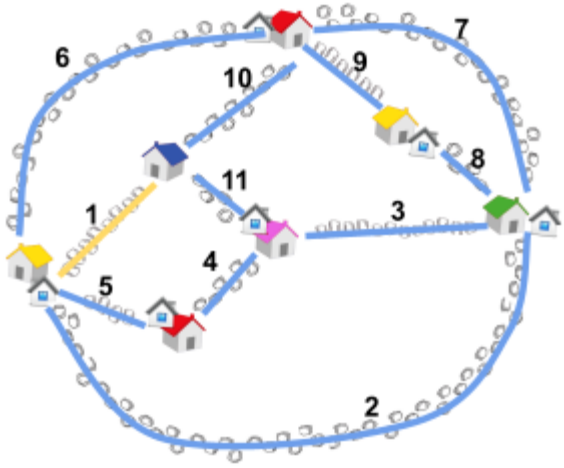




3. Find a **Hamiltonian** circuit (start and finish at the green house) **There are multiple answers, this is just one example.**



4. To find the street names you need an **Eulerian Path.**
5. Find an Eulerian circuit. **There are a few answers, this is just one.**



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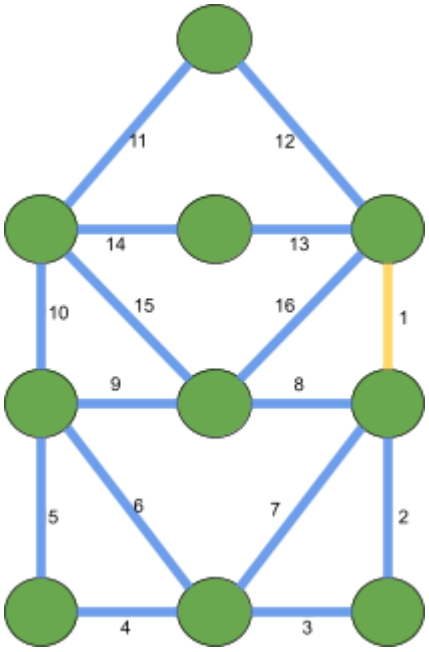
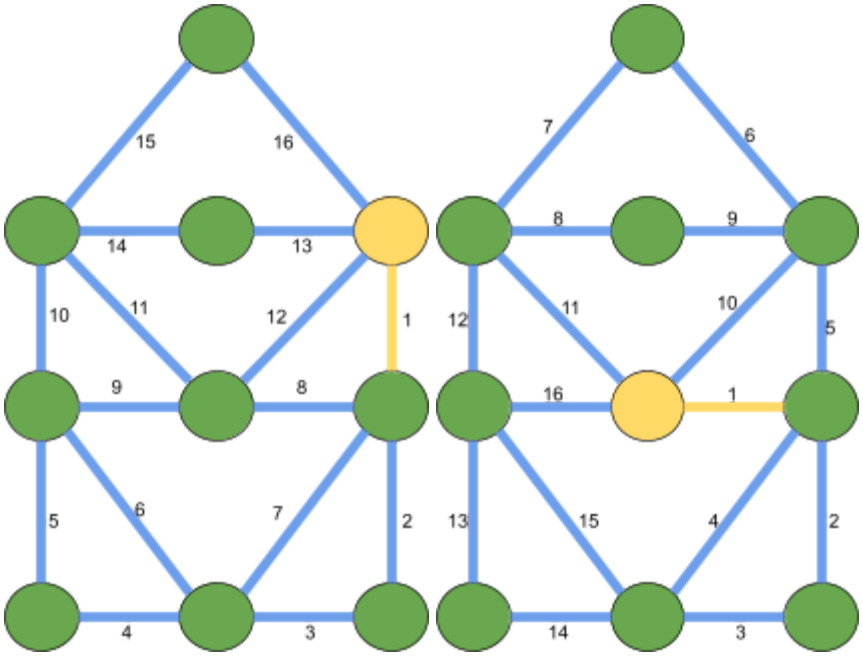
Puzzle 2!

1. You need to find an **Eulerian Path.**
2. Find An Eulerian Path. **There are multiple solutions, this is just one example**



3. Find an Eulerian Circuit. **There are multiple answers, these are 2 examples.**



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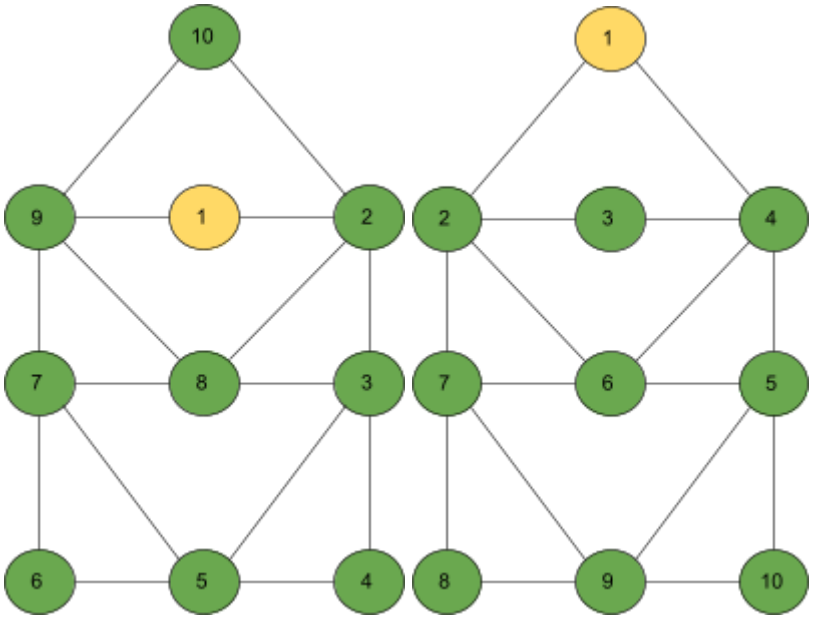
4. **A Hamiltonian Path** reaches each of the nodes.
5. Find a Hamiltonian Path. **There are multiple answers, this is just two examples.**



6. Find a Hamiltonian Circuit.
   This graph does not have one.
   There is no way to go to each node exactly once and get back to where you started.

Puzzle 3!

1. An Eulerian Path
2. Find a path. **There are multiple answers, this is just one example.**

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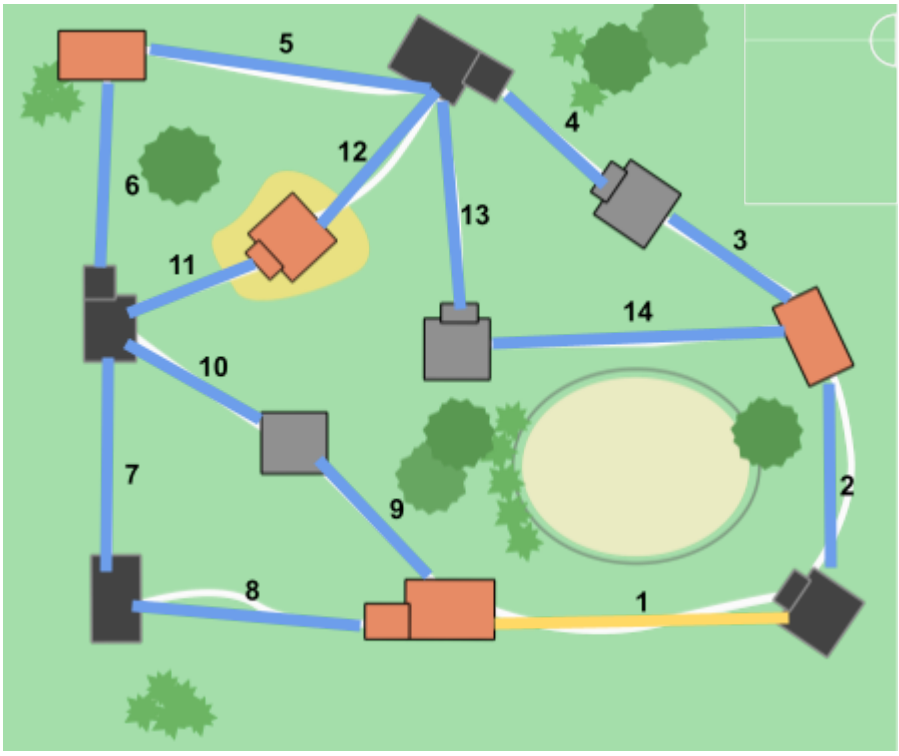

3. Find a circuit
   This graph does not have an Eulerian Circuit. There is no way to go to each edge exactly once and get back to where you started.

4. What kind of Path are you looking for? **A Hamiltonian Path.**
5. Can you find a path?
   This graph does not have an Hamiltonian Path.
   There is no way to go to each node exactly once and get back to where you started.
   This means it cannot have a Hamiltonian Circuit either.

Bonus Eulerian Puzzle.

- An Eulerian Circuit is only possible if **every node has an even number of edges.**
- An eulerian Path is only possible if **every node has an even number of edges or exactly 2 nodes have an odd number of edges.**

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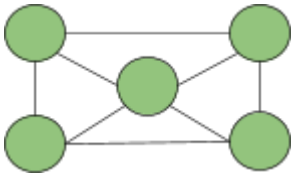
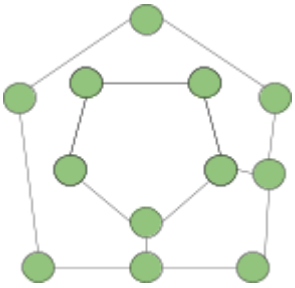
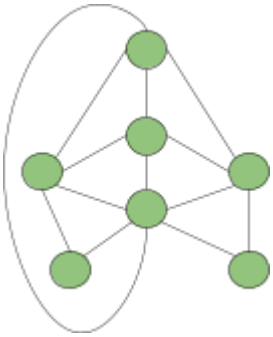
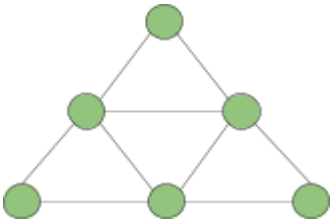
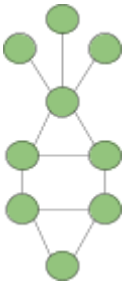
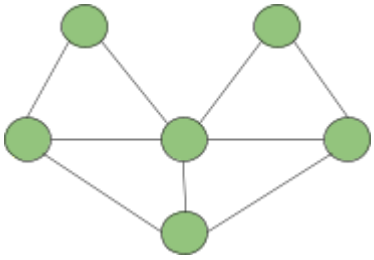
 **Path?: No** **Circuit? No**	 **Path?: No** **Circuit? No**	 **Path?: Yes** **Circuit? Yes**
 **Path?: Yes** **Circuit? Yes**	 **Path?: No** **Circuit? No**	 **Path?: No** **Circuit? No**

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This page is for

Teachers

Want more?

Here are some further activities, online resources, assessment ideas and curriculum references.

Keep the conversation going

- We only worked with undirected graphs in this activity. Can you think of a real world example of a Directed graph?
- Try drawing your own directed graph and solving as many paths and circuits as you can. If you can't find any paths at all, adjust your directions and try again.

Keep learning

For students who would like to try designing their own paths online, there is a graph construction tool that will also help you find paths and circuits:
https://graphonline.ru/en/

Students interested in exploring a theorem for determining if a graph has a Hamiltonian Path can look into Ore's Theorem.
A good introduction can be found at:
https://study.com/academy/answer/how-to-determine-if-the-graph-is-hamiltonian.html

Resources

(7-8) Students use physical characteristics of different animals to develop an algorithm that allows you to easily group and identify each animal based on a series of simple questions:
cmp.ac/classifying

(7-8, 9-10) Students learn about Voroni algorithms and how they are used to determine with certainty the shortest distance to key locations on a map:
cmp.ac/stores

For teachers creating a portfolio of learning or considering this task for assessment

Understanding that the graphs represent real world situations is important. Have students come up with their own scenarios and create graphs that correspond.

Or, present students with maps containing different nodes (houses, hospitals, bus stops, train stations) and ask them to construct graphs of their connections. For example, the graph of all the bus stop locations in a suburb and the roads between them.

To test understanding of the Eulerian circuit and path rule from puzzle 3, present students with graphs that do not have circuits or paths and ask them to adjust the graph so that the circuit becomes possible. Ask them to explain the change they made in relation to the rule.

Linking it back to the Australian Curriculum: Digital Technologies

Algorithms
Design algorithms represented diagrammatically and in English, and trace algorithms to predict output for a given input and to identify errors.
(ACTDIP019 / ACTDIP029 - see cmp.ac/algorithms)

Refer to aca.edu.au/curriculum for more curriculum information.

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